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Spontaneous symmetry breaking in a two-lane system with parallel update

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Abstract
This paper studies a two-lane totally asymmetric simple exclusion processes (TASEP) with narrow entrances under parallel update. Particles move on two parallel lanes in the opposite directions without lane changing. The narrow entrances are modelled in this way: the entry of a particle is not allowed if the exit site of the other lane is occupied (see also Pronina and Kolomeisky 2007 *J. Phys. A: Math. Theor.* **40** 2275). We mainly focus on the case where particles hop deterministically in the bulk. The phase diagram, bulk density and particle currents are analysed using mean-field approximation. It is shown that there are two symmetry breaking phases, and one of them (i.e., asymmetric low density/low-density phase) occupies only a line in the phase diagram. A multi-stable phenomenon is also observed. Monte Carlo simulations are carried out and the simulation results deviate from the mean-field prediction, because correlations are not considered in mean-field calculation. A seesaw phase is reproduced when $\alpha = 1$. The results are also compared with that obtained from the two-lane system with random update. Finally, preliminary simulation results where particles hop with the rate $q < 1$ in the bulk are reported and it is shown that the introduction of stochastic hopping changes the phase diagram structure.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

In recent years, driven diffusive systems have attracted the interests of physicists because they show a variety of nonequilibrium effects [1, 2]. A very prominent example is the asymmetric

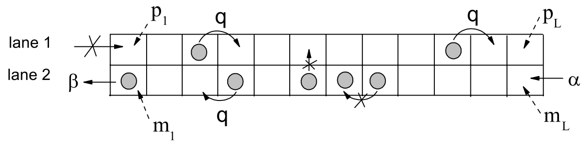

Figure 1. Sketch of the model. On lane 1, particles move from left to right and on lane 2, particles move from right to left. The entrance site on lane 1 is denoted by p_1 and the exit site on lane 1 is denoted by p_L. The entrance site on lane 2 is denoted by m_L and the exit site on lane 2 is denoted by m_1. In the bulk, the particles hop forward with rate q provided the target site is empty.

simple exclusion processes (ASEPs), which are discrete non-equilibrium models that describe the stochastic dynamics of multi-particle transport along one-dimensional lattices. Each lattice site can either be empty or occupied by a single particle. Particles interact only through hard-core exclusion potential. ASEPs were introduced in 1968 as theoretical models for describing the kinetics of biopolymerization [3] and have been applied successfully to understand polymer dynamics in dense media [4], diffusion through membrane channels [5], gel electrophoresis [6], dynamics of motor proteins moving along rigid filaments [7], the kinetics of synthesis of proteins [8] and traffic flow analysis [9, 10].

Many non-equilibrium behaviours such as boundary-induced phase transition, the unusual dynamical scaling and spontaneous symmetry breaking have been observed in ASEPs. The first model that exhibits spontaneous symmetry breaking was a model with open boundaries and it is known as the 'bridge model' [11]. In the model, two species of particles move in the opposite directions. It was shown that two phases with broken symmetry could exist, despite the update rules are symmetric with respect to the two species. While it is argued by Monte Carlo simulations that one of the phases (i.e., asymmetric low density/low-density phase) may not exist in the thermodynamic limit [12–18], the mean-field analysis shows that it could exist in a very small region.

To analyse more realistic phenomena, the multi-lane ASEPs have been developed to model the transport along parallel channels [18–23]. Recently, Pronina and Kolomeisky [24] have studied the spontaneous symmetric breaking in a two-lane ASEP with narrow entrances. It is shown that there are two phases exhibiting spontaneous symmetric breaking as in the 'bridge model'.

In most symmetry breaking models, random update rules are adopted, which introduce stochastic effects not only at boundaries but also in the bulk. Willmann *et al* [15] investigated the bridge model with parallel sublattice update and it is shown that the symmetric breaking is due to an amplification mechanism of fluctuations.

In this paper, we investigate a two-lane model with narrow entrances under parallel update rules. We study the system using both mean-field approach and extensive Monte Carlo simulations. The results are compared with those obtained from random update rules.

The paper is organized as follows. In section 2, we give a brief description of the model. In section 3, we present the results of mean-field calculations. In section 3, we discuss the results of MC simulations. We give our conclusions in section 4.

2. Model

The two-lane model with narrow entrances is proposed by Pronina and Kolomeisky [24]. The system consists of two parallel one-dimensional L lattices with two species of particles moving in different lanes in the opposite directions (see figure 1). The hoppings between the lanes are

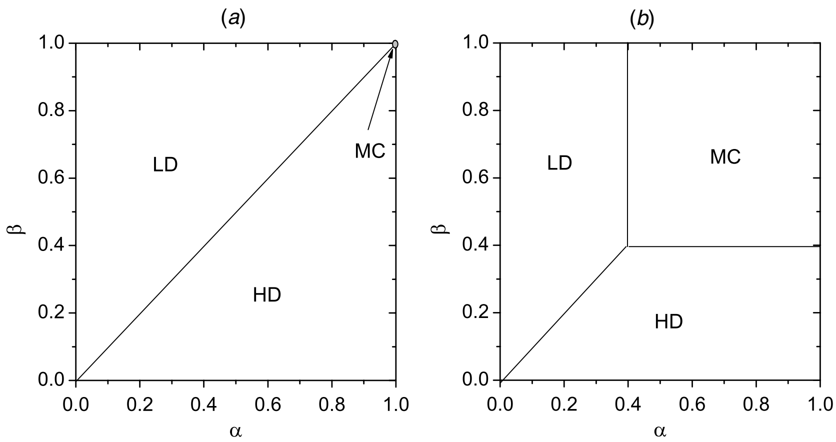

Figure 2. Phase diagram of single lane ASEP under open boundary conditions. (*a*) $q = 1$; (*b*) $q < 1$.

not allowed. In the bulk, a particle hops to the next site with probability q provided the target site is empty. At the exit site, a particle is removed with probability β. At the entrance site, a particle is injected with probability α provided this site and exit site on the other lane are empty. In our model, the parallel update rules are adopted, i.e., the dynamics is applied to all sites at the same time. In contrast, in random update rules adopted in [24], a site is chosen randomly in each time step and one Monte Carlo step includes $2L$ time steps.

In this paper, we mainly discuss the case $q = 1$, i.e., the dynamics are deterministic in the bulk. A preliminary simulation of $q < 1$ is also presented and the detailed investigations will be carried out in future work.

3. Mean-field calculation

In this section, we present the mean-field calculation in the case of $q = 1$. First, let us briefly recall the results of ASEP on a single lane with open boundaries. For $q = 1$, when entrance probability is larger than removal probability (i.e., $\alpha > \beta$), the system is in a high-density (HD) phase and the bulk density is $\rho = \frac{1}{\beta+1}$, the current is $J = \frac{\beta}{\beta+1}$. When $\alpha < \beta$, the system is in a low-density (LD) phase and the bulk density is $\rho = \frac{\alpha}{\alpha+1}$, the current is $J = \frac{\alpha}{\alpha+1}$. When $\alpha = \beta$, domain wall separating high-density region and low-density region performs a random walk and a linear density profile appears. The maximum current (MC) $J = 0.5$ can only be reached at $\alpha = \beta = 1$. In this case, the dynamics is completely deterministic and a particle is injected into the system every two time steps and similarly, a particle is removed from the system every two time steps (see figure 2(*a*)) [25, 26].

For $q < 1$, the MC phase begins to expand in the phase diagram: it exists when $\alpha > 1-\sqrt{1-q}$ and $\beta > 1-\sqrt{1-q}$ and the maximum current is $J = \frac{1}{2}(1-\sqrt{1-q})$. When $\alpha > \beta$ and $\beta < 1-\sqrt{1-q}$, the system is in the HD phase and the current is $J = \frac{\beta(q-\beta)}{q-\beta^2}$. When $\alpha < \beta$ and $\alpha < 1-\sqrt{1-q}$, the system is in the LD phase and the current is $J = \frac{\alpha(q-\alpha)}{q-\alpha^2}$ (see figure 2(*b*)).

Next we consider the two-lane ASEP with narrow entrances. First, we consider the special case $\alpha = \beta = 1$. For simplicity, we assume initially there is no particle in the system. In this case, two subcases are distinguished.

- *L is odd.* Let us introduce occupation variables p_i and m_i for lanes 1 and 2, so that $p_i = 1$ and $m_i = 1$ if corresponding site i is occupied, and 0 if unoccupied. In this subcase, it is obvious that $p_1 = m_1$ and $m_L = p_L$ are always met after the first particle reaches exit site. In other words, the exit site on the other lane will be empty if the entrance site is empty and vice versa. Therefore, the narrow entrances have no effect on the dynamics of the system. The maximum current can be reached in both lanes.
- *L is even.* In this subcase, it can easily be found that the current depends on the system size as $J = \frac{N/2}{N+1}$. Therefore, the maximum current can be achieved only when $N \to \infty$.

When $\alpha \neq 1$ or $\beta \neq 1$, the maximum current phase cannot be reached in both lanes. Therefore, next we discuss the five situations, i.e., asymmetric HD/HD phase, asymmetric HD/LD phase, asymmetric LD/LD phase, symmetric HD phase and symmetric LD phase.

It can easily be understood that the asymmetric the HD/HD phase does not exist, because the situation on each lane is determined by β in HD phase and β is the same for both lanes.

For the other cases, let us assume the effective entrance rates are given by α_1 and α_2. Therefore

$$\alpha_1 = \alpha(1 - m_1) \qquad \text{and} \qquad \alpha_2 = \alpha(1 - p_L). \tag{1}$$

3.1. Asymmetric HD/LD phase

First, we consider the asymmetric HD/LD phase. Without loss of generality, we assume that lane 1 is in HD and lane 2 is in LD. This is fulfilled when

$$\alpha_1 > \beta \qquad \text{and} \qquad \alpha_2 < \beta. \tag{2}$$

Since lane 1 is in HD, the bulk density in lane 1 is

$$\rho_1 = \frac{1}{\beta + 1}. \tag{3}$$

The density on the exit site L equals the bulk density, so that

$$p_L = \frac{1}{\beta + 1}. \tag{4}$$

Thus,

$$\alpha_2 = \alpha\left(1 - \frac{1}{\beta + 1}\right) = \frac{\alpha\beta}{\beta + 1}. \tag{5}$$

Since lane 2 is in LD, the current is

$$J_2 = \frac{\alpha_2}{\alpha_2 + 1} = \frac{\frac{\alpha\beta}{\beta+1}}{\frac{\alpha\beta}{\beta+1} + 1} = \frac{\alpha\beta}{\alpha\beta + \beta + 1}. \tag{6}$$

The bulk density in lane 2 is

$$\rho_2 = \frac{\alpha\beta}{\alpha\beta + \beta + 1}. \tag{7}$$

The current in lane 2 can also be calculated by

$$J_2 = m_1\beta. \tag{8}$$

Thus, from equations (6) and (8), one can obtain

$$m_1 = \frac{\alpha}{\alpha\beta + \beta + 1}. \tag{9}$$

Therefore,

$$\alpha_1 = \alpha(1 - m_1) = \alpha\left(1 - \frac{\alpha}{\alpha\beta + \beta + 1}\right). \tag{10}$$

From equations (2) and (5), we have

$$\frac{\alpha\beta}{\beta + 1} < \beta, \tag{11}$$

which is always met. From equations (2) and (10), one has

$$\alpha\left(1 - \frac{\alpha}{\alpha\beta + \beta + 1}\right) > \beta. \tag{12}$$

Solving equation (12) (see the appendix), one has

$$\frac{\beta}{1 - \beta} < \alpha < 1 + \beta \qquad \text{for} \quad \beta < \frac{\sqrt{5} - 1}{2} \tag{13}$$

and

$$1 + \beta < \alpha < \frac{\beta}{1 - \beta} \qquad \text{for} \quad \beta > \frac{\sqrt{5} - 1}{2}. \tag{14}$$

It is obvious that equation (14) can never be met and the right-hand side of equation (13) can always be met. Therefore, for the HD/LD phase to exist, one has

$$\frac{\beta}{1 - \beta} < \alpha \leqslant 1. \tag{15}$$

3.2. *Asymmetric LD/LD phase*

Next we discuss the asymmetric LD/LD phase, which exists if

$$\alpha_1 < \beta \qquad \text{and} \qquad \alpha_2 < \beta \qquad \text{and} \qquad \alpha_1 \neq \alpha_2. \tag{16}$$

Since both lanes are in LD, we have

$$J_1 = \frac{\alpha_1}{\alpha_1 + 1}, \qquad J_2 = \frac{\alpha_2}{\alpha_2 + 1}. \tag{17}$$

J_1 and J_2 can also be calculated by

$$J_1 = p_L\beta \qquad \text{and} \qquad J_2 = m_1\beta. \tag{18}$$

Therefore, we have

$$p_L = \frac{\alpha_1}{(\alpha_1 + 1)\beta} \qquad \text{and} \qquad m_1 = \frac{\alpha_2}{(\alpha_2 + 1)\beta}. \tag{19}$$

Therefore, α_1 and α_2 are determined by

$$\alpha_1 = \alpha(1 - m_1) = \alpha\left(1 - \frac{\alpha_2}{(\alpha_2 + 1)\beta}\right), \tag{20}$$

$$\alpha_2 = \alpha(1 - p_L) = \alpha\left(1 - \frac{\alpha_1}{(\alpha_1 + 1)\beta}\right). \tag{21}$$

Substituting equation (20) into equation (21), we have

$$\alpha_2 = \alpha\left[\frac{\alpha\beta(\alpha_2\beta + \beta - \alpha_2) + \beta(\alpha_2\beta + \beta) - \alpha(\alpha_2\beta + \beta - \alpha_2)}{\alpha\beta(\alpha_2\beta + \beta - \alpha_2) + \beta(\alpha_2\beta + \beta)}\right]. \tag{22}$$

From equation (22), one can obtain

$$(\alpha\beta - \alpha + \beta)\left[\beta\alpha_2^2 + (\alpha + \beta - \alpha\beta)\alpha_2 - \alpha\beta\right] = 0. \tag{23}$$

Equation (23) can be met if

$$\alpha\beta - \alpha + \beta = 0 \tag{24}$$

or

$$\beta\alpha_2^2 + (\alpha + \beta - \alpha\beta)\alpha_2 - \alpha\beta = 0. \tag{25}$$

When equation (24) is met, i.e., $\alpha = \frac{\beta}{1-\beta}$, equation (20) becomes

$$\alpha_1 = \frac{\beta}{1-\beta}\left(1 - \frac{\alpha_2}{(\alpha_2 + 1)\beta}\right). \tag{26}$$

Since LD/LD requires $\alpha_1 < \beta$, equation (26) becomes

$$\frac{1}{1-\beta}\left(1 - \frac{\alpha_2}{(\alpha_2 + 1)\beta}\right) < 1, \tag{27}$$

which leads to

$$\alpha_2 > \frac{\beta^2}{1-\beta^2}. \tag{28}$$

This means, there are multi-values of α_2 corresponding to a pair of (α, β), provided $\frac{\beta^2}{1-\beta^2} < \alpha_2 < \beta$. Once α_2 is given, α_1 can be calculated from equation (20). For most values of $\alpha_2, \alpha_1 \neq \alpha_2$, but for a special value of $\alpha_2, \alpha_1 = \alpha_2$. (This value of α_2 is obtained by setting $\alpha_1 = \alpha_2$ in equation (20).) In other words, there are multi-LD/LD phases and one symmetric phase corresponding to a pair of (α, β).

When equation (25) is met, α_2 can be solved,

$$\alpha_2 = \frac{\alpha\beta - \alpha - \beta \pm \sqrt{(\alpha + \beta - \alpha\beta)^2 + 4\alpha\beta^2}}{2\beta}. \tag{29}$$

Since $\alpha_2 > 0$, the solution related to '$-$' needs to be discarded. Substituting $\alpha_2 = \frac{\alpha\beta-\alpha-\beta+\sqrt{(\alpha+\beta-\alpha\beta)^2+4\alpha\beta^2}}{2\beta}$ into equation (20), one has $\alpha_1 = \alpha_2$. Therefore, LD/LD does not exist even when equation (25) is met.

3.3. Symmetric LD phase

The symmetric LD phase exists if

$$\alpha_1 < \beta \qquad \text{and} \qquad \alpha_2 = \alpha_1. \tag{30}$$

Substituting $\alpha_2 = \alpha_1$ into equation (20), one has

$$\alpha_1 = \alpha\left(1 - \frac{\alpha_1}{(\alpha_1 + 1)\beta}\right). \tag{31}$$

Reformulating equation (31), one has

$$\beta\alpha_1^2 + (\alpha + \beta - \alpha\beta)\alpha_1 - \alpha\beta = 0, \tag{32}$$

which is identical to equation (25). The solution is, therefore, $\alpha_1 = \alpha_2 = \frac{\alpha\beta-\alpha-\beta+\sqrt{(\alpha+\beta-\alpha\beta)^2+4\alpha\beta^2}}{2\beta}$. Together with $\alpha_1 < \beta$, we have

$$\alpha < \beta + 1. \tag{33}$$

Equation (33) can always be met. Therefore, the phase boundary between HD/LD and symmetric LD is determined by $\alpha = \frac{\beta}{1-\beta}$.

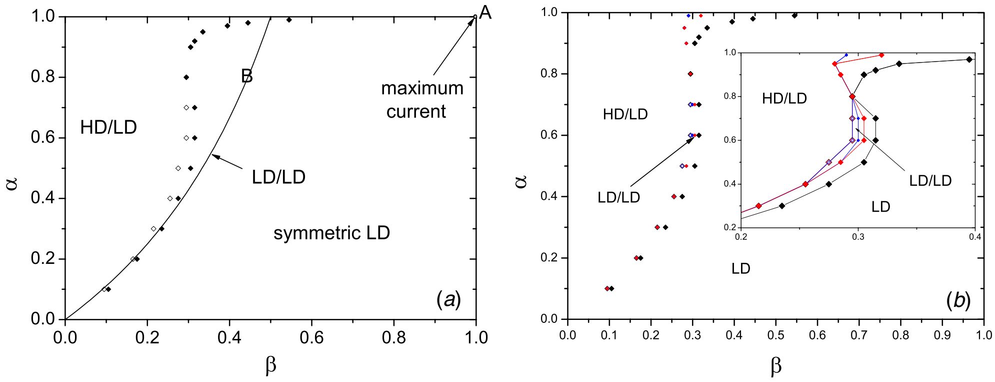

Figure 3. Phase diagram of the system. For $L = 5000$, a transient 10^7 time steps are discarded and we gather data for 4×10^7 time steps. Black symbols for $L = 100$, red symbols for $L = 1000$ and blue symbols for $L = 5000$. In (a), the solid line is from mean-field calculations; in (b), the inset shows the details for $0.2 < \beta < 0.4$.

3.4. Symmetric HD phase

Finally, let us check symmetric HD phase. Since lane 1 is in HD, the bulk density in lane 1 is

$$\rho_1 = \frac{1}{\beta + 1}. \tag{34}$$

The density on the exit site L equals the bulk density, so that

$$p_L = \frac{1}{\beta + 1}. \tag{35}$$

Therefore

$$\alpha_2 = \alpha\left(1 - \frac{1}{\beta + 1}\right) = \frac{\alpha\beta}{\beta + 1}. \tag{36}$$

Together with $\alpha_2 > \beta$, we have

$$\frac{\alpha\beta}{\beta + 1} > \beta. \tag{37}$$

This will never be met. Therefore, symmetric HD phase does not exist.

The phase diagram predicted by mean-field calculation is shown in figure 3(a). Different from the results in [24], the maximum current phase only exists at a single point A, and the LD/LD phase exists on a line B instead of in a small region. Furthermore, the existence of multi-phases on line B and at point A is observed.

4. Simulation results

In this section, the Monte Carlo simulation results are presented. First, we consider $q = 1$. We consider three typical system of size $L = 100$, 1000 and 5000. Following [16], we investigate the particle density histograms $P_L(\rho_1, \rho_2)$, where ρ_1 and ρ_2 are instantaneous densities of particles in lane 1 and in lane 2,respectively. We focus on the observables $\rho_{\min}$ and $\rho_{\max}$, denoting the smaller one ($\rho_{\min}$) and larger one ($\rho_{\max}$) of ρ_1 and ρ_2 at the global maximum of $P_L(\rho_1, \rho_2)$. When $\rho_{\max} = \rho_{\min} < 0.5$, the system is in symmetric LD phase; when $\rho_{\max} = \rho_{\min} > 0.5$, the system is in symmetric HD phase; when $\rho_{\max} = \rho_{\min} = 0.5$, the

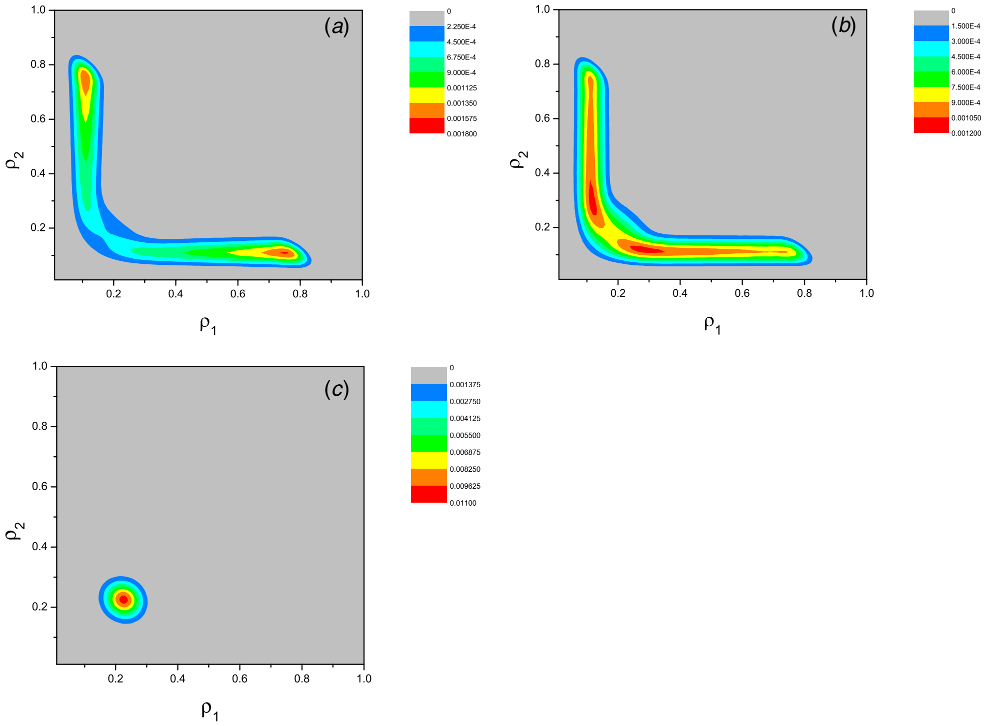

Figure 4. Particle density histogram at $\alpha = 0.5$. (*a*) $\beta = 0.27$, HD/LD phase; (*b*) $\beta = 0.28$, asymmetric LD/LD phase; (*c*) $\beta = 0.5$, symmetric LD phase.

system is in symmetric MC phase; when $\rho_{\min} < \rho_{\max} < 0.5$, the system is in asymmetric LD/LD phase; when $\rho_{\max} > 0.5$ and $\rho_{\min} < 0.5$, the system is in asymmetric HD/LD phase

First system size $L = 100$ is considered. Initially, the system is empty, and a transient time of 10^8 time steps is discarded. We gather data for 4×10^8 time steps unless otherwise mentioned.

Figure 4 shows three typical particle density histograms in the HD/LD phase, asymmetric LD/LD phase and symmetric LD phase. One can see that in the symmetric LD phase, a single peak exists on the diagonal, while in asymmetric phases, a double peak with two off-diagonal maxima appears. As indicated in [16], the transition between the two asymmetric phases is marked by histograms with two long ridges: one running close to the ρ_1 axis and the other close to ρ_2 axis. At the transition, the global maximum shifts from the near end to the far end of each ridge.

Figure 5 shows $\rho_{\min}$ and $\rho_{\max}$ against β with α fixed. One can see that for $\alpha = 0.2$ and 0.5, three phases are distinguished. The asymmetric LD/LD phase appears if β is between the values shown by the two dashed lines. When β is smaller than the value shown by the left dashed line, a HD/LD phase occurs. When β is larger than the value shown by the right dashed line, a symmetric LD phase occurs. In contrast, for $\alpha = 0.8$ and 0.99, the asymmetric LD/LD phase does not exist.

Nevertheless, we need to point out that the situation is quite different for $\alpha = 1$ (see figure 6). One can see that the global maximum could be achieved along the line $\rho_1 + \rho_2 = 1$. With the increase (decrease) of β, the global maximum value increases (decreases) and the

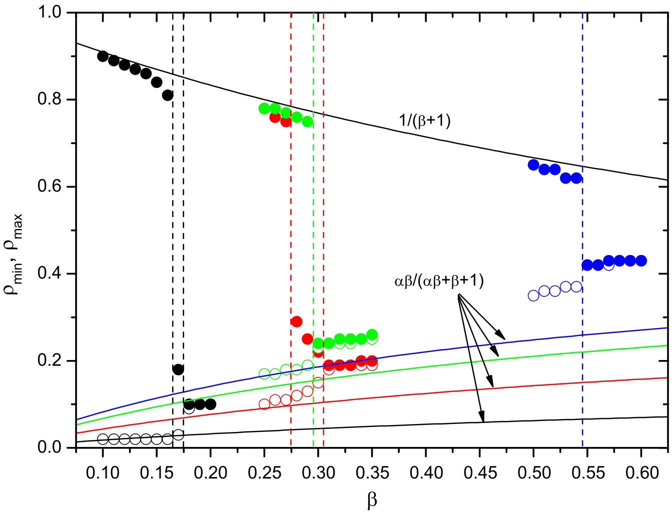

Figure 5. $\rho_{\min}$ and $\rho_{\max}$ against β. The black symbols correspond to $\alpha = 0.2$, red symbols to $\alpha = 0.5$, green symbols to $\alpha = 0.8$ and blue symbols to $\alpha = 0.99$. The filled (open) symbols show $\rho_{\max}$ ($\rho_{\min}$). The solid lines are mean-field predictions. The upper line is from equation (3) and the lower lines are from equation (7). One can see the mean-field predictions are in agreement with simulation results for $\rho_{\max}$. For $\rho_{\min}$, the mean-field predictions are in agreement with simulation results when α is small and there exists large deviation when α is large.

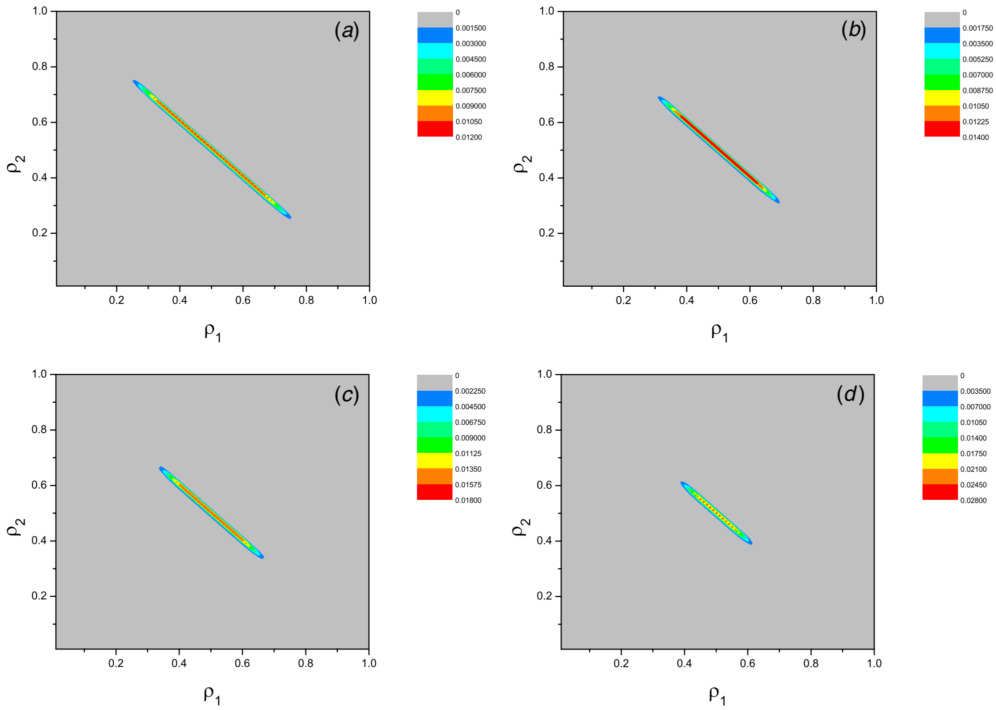

Figure 6. Particle density histogram showing seesaw phase, $\alpha = 1$. (*a*) $\beta = 0.4$; (*b*) $\beta = 0.52$; (*c*) $\beta = 0.57$; (*d*) $\beta = 0.7$.

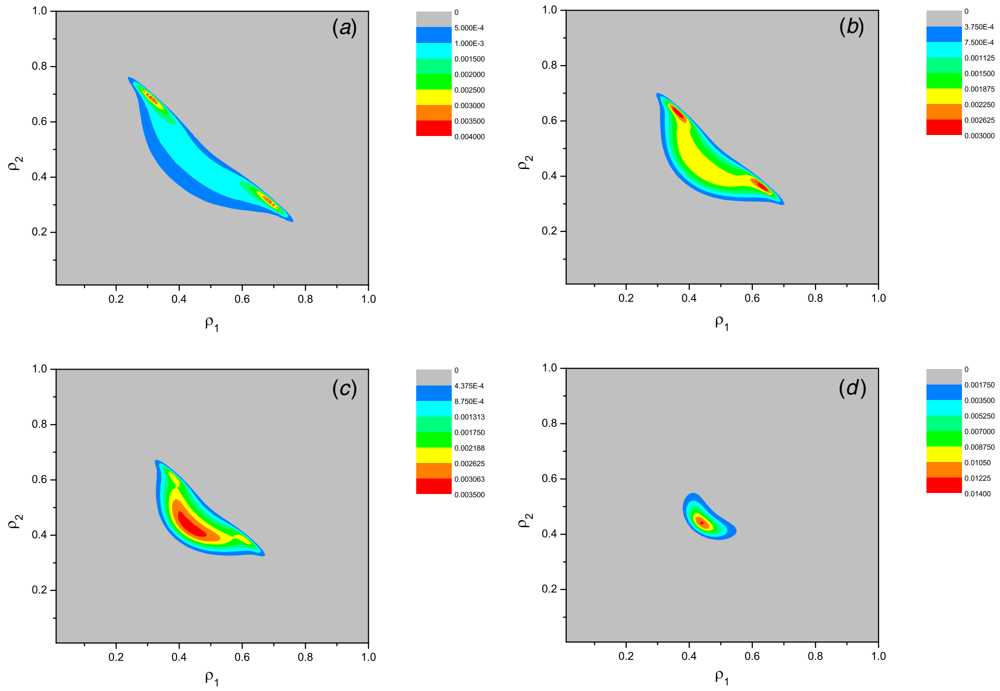

Figure 7. Particle density histogram for $\alpha = 0.99$. (*a*) $\beta = 0.4$, HD/LD phase; (*b*) $\beta = 0.52$, HD/LD phase; (*c*) $\beta = 0.57$, symmetric LD phase; (*d*) $\beta = 0.7$, symmetric LD phase.

region that global maximum could be achieved shrinks (expands). This phase is named 'seesaw phase' in [18]. In contrast, even if α is slightly smaller than 1, the particle density histogram will be quite different (cf figure 7).

The phase diagram is also shown in figure 3(*a*). One can see that the phase structure from Monte Carlo simulation deviates from the mean-field result. This is because, as indicated in [24], correlations are important in the dynamics of this system, especially for parallel update rules, which usually produce the strongest correlations among different update rules [27]. This perhaps could also partially explain why the results are different from that obtained by random update rules.

It is also qualitatively different from that obtained from random update (cf figure 2 in [24]): (i) asymmetric LD/LD phase does not exist for large value of α; (ii) the boundary between HD/LD phase and symmetric LD phase bends downwards when α approaches one; (iii) the maximum current phase only exists on a point and its existence depends on the system size L.

With the increase of system size, the results remains qualitatively unchanged. A quantitative change of the phase boundaries occurs (figure 3(*b*)). The region of asymmetric LD/LD phase shrinks. Note that the boundary between HD/LD phase and asymmetric LD/LD phase essentially does not depend on the system size, but the boundary between asymmetric LD/LD phase and symmetric LD phase shifts left. The boundary between HD/LD phase and symmetric LD phase also shifts left. In the thermodynamic limit, the LD/LD phase may disappear, but this needs further investigations.

Next the preliminary simulation results for $q < 1$ are presented. It is found with the introduction of stochastic hopping in the bulk, (i) the seesaw phase disappears (figure 8(*a*))

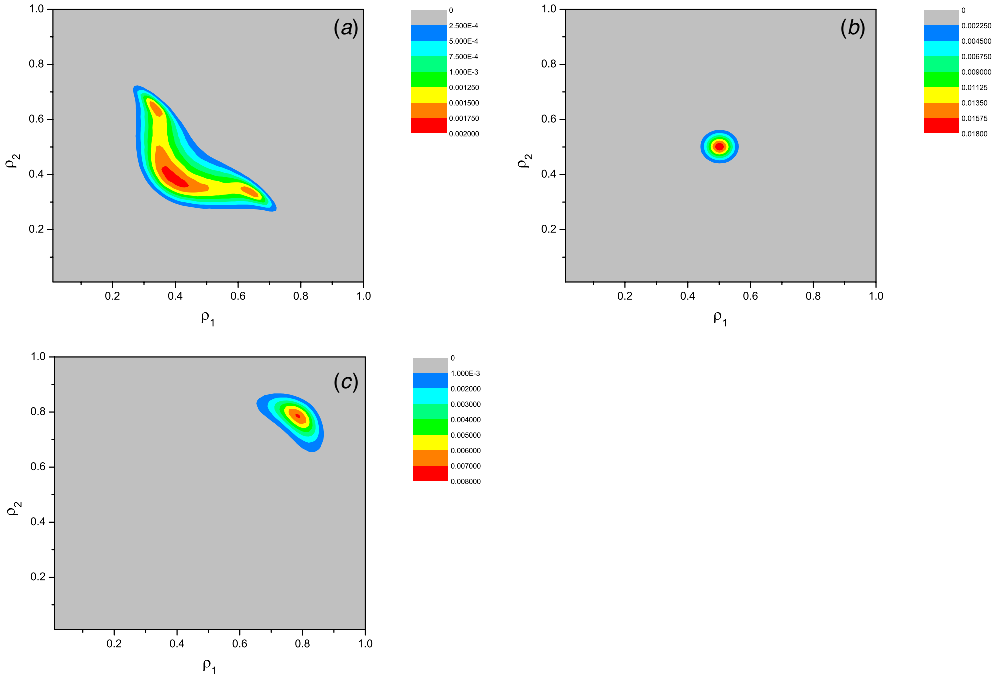

Figure 8. Particle density histogram with the introduction of stochastic hopping in the bulk. (a) $\alpha = 1.0, \beta = 0.4, q = 0.9$, the seesaw phase disappears and symmetric LD phase appears instead; (b) $\alpha = 1.0, \beta = 0.5, q = 0.5$, symmetric MC phase; ($c$) $\alpha = 1.0, \beta = 0.1, q = 0.5$, symmetric HD phase.

and (ii) the MC/MC phase expands (figure 8(b)). Furthermore, a symmetric HD phase begins to appear (figure 8(c)). In our future work, more details of effect of stochastic hopping on symmetry breaking will be investigated.

5. Conclusion

In this paper, we have studied a two-lane totally asymmetric simple exclusion process (TASEP) with narrow entrances under parallel update. First, the system in which particles deterministically hop in the bulk is considered. Mean-field analysis shows two symmetry breaking phases appear. However, different from previous results obtained under random update rules, the asymmetric LD/LD phase is found to occupy only one line (i.e., line B) in the phase diagram. Furthermore, a multi-stable phenomenon is observed. Specifically, (i) on line B, both asymmetric LD/LD phase and symmetric LD phase could exist; (ii) on point A (i.e., $\alpha = \beta = 1$), the maximum current phase could appear if system size L is odd. In contrast, if L is even, the current depends on the system size and a maximum current can appear only when $N \to \infty$.

Next Monte Carlo simulations are carried out, and particle density histograms $P_L(\rho_1, \rho_2)$ are investigated. It is shown the phase structure from Monte Carlo simulation deviates from the mean-field result. This may be because correlations are important in the dynamics of this system. In the simulations, a seesaw phase is reproduced when $\alpha = 1$. It is also found that the asymmetric LD/LD phase occupies a small region instead of a line in the phase diagram when the system size L is small. With the increase of L, the region shrinks. However, presently we could not conclude that in the thermodynamic limit, the LD/LD phase will disappear. This needs further investigations. If possible, we will try to carry out some exact analysis of the problem, using matrix product method and/or Bethe Ansatz method.

We also compare the results with that obtained from random update. The differences are (i) asymmetric LD/LD phase does not exist for large value of α; (ii) for small system size, the boundary between the HD/LD phase and symmetric LD phase bends downward when α approaches one; (iii) the maximum current phase only exists on a point and its existence depends on the system size L.

The system in which particles hop with rate $q < 1$ in the bulk is also studied. It is found the introduction of stochastic hopping in the bulk has changed the phase diagram structure of the system, the seesaw phase disappears and the symmetric HD phase appears.

Our works show that update rules have played an important role in the spontaneous symmetry breaking phenomenon. The original bridge model will also need to be investigated with parallel update rules, and this work is now in progress and will be reported in future publications.

Acknowledgments

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Appendix

In this appendix, we solve equation (12)

$$\alpha\left(1-\frac{\alpha}{\alpha\beta+\beta+1}\right) > \beta. \tag{A.1}$$

From equation (A.1), we have

$$\alpha\left(\frac{\alpha\beta+\beta-\alpha+1}{\alpha\beta+\beta+1}\right) > \beta, \tag{A.2}$$

which becomes

$$\alpha(\alpha\beta+\beta-\alpha+1) > (\alpha\beta+\beta+1)\beta, \tag{A.3}$$

i.e.,

$$\alpha^2\beta+\alpha\beta-\alpha^2+\alpha > \alpha\beta^2+\beta^2+\beta. \tag{A.4}$$

Therefore, we have

$$(1-\beta)\alpha^2+(\beta^2-\beta-1)\alpha+\beta^2+\beta < 0. \tag{A.5}$$

The two solutions of the equation $(1-\beta)\alpha^2+(\beta^2-\beta-1)\alpha+\beta^2+\beta = 0$ are

$$\alpha_{1,2} = \frac{1+\beta-\beta^2 \pm \sqrt{(\beta^2-\beta-1)^2-4(1-\beta)(\beta^2+\beta)}}{2(1-\beta)}. \tag{A.6}$$

Since $(\beta^2-\beta-1)^2-4(1-\beta)(\beta^2+\beta)=\beta^4+2\beta^3-\beta^2-2\beta+1=(1-\beta-\beta^2)^2$, we have

$$\alpha_{1,2}=\frac{1+\beta-\beta^2\pm|(1-\beta-\beta^2)|}{2(1-\beta)}. \tag{A.7}$$

Thus $\alpha_1=1+\beta$ and $\alpha_2=\frac{\beta}{1-\beta}$. When $\beta>\frac{\sqrt{5}-1}{2}, \alpha_1<\alpha_2$; when $\beta<\frac{\sqrt{5}-1}{2}, \alpha_1>\alpha_2$. Therefore, we have equations (13) and (14).

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